Crane.
Investigations on
magnetic fields with
reference to ore-concentration.

INVESTIGATIONS

ON

Magnetic Fields with Reference to Ore-Concentration.

BY

W. R. CRANE.

SUBMITTED IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR
THE DEGREE OF DOCTOR OF PHILOSOPHY IN THE FACULTY
OF APPLIED SCIENCE, COLUMBIA UNIVERSITY.

NEW YORK.
1900.

Investigations of Magnetic Fields, with Reference to Ore-Concentration.*

BY WALTER R. CRANE, LAWRENCE, KANSAS.

(Richmond Meeting, February, 1901.)

CONTENTS.

INTRODUCTION.

THE form and strength of magnetic fields have attracted much attention, especially in connection with the designing of dynamo-electric machinery—an art which has been revolution-

---

ized within a comparatively short period by the results of scientific research in this field.

Summarizing the known facts, we may say that the form and strength of magnetic fields are controlled: first, by the form, size and quality of the material composing the magnetic circuit; second, by the ampère-turns of the exciting current; third, by the distance between the poles; and fourth, by the shape of pole-pieces. The present paper has to do with the last two factors, pole-distance and pole-form, only; but in order that the experiments here reported may be clearly understood, the apparatus and methods employed will be first described.

I. APPARATUS AND METHODS.

1. *The Magnetic Circuit.*

The general arrangement of the magnetic circuit and its connections is shown in Fig. 1, in which M is the magnet employed in testing the minerals; R, the resistance in series with the magnet M; W, the four wires connecting with the main switch-board; R-S, the switch that reverses the current in M; T, the test-coil used in determining the amount of magnetization in M; G, the galvanometer used to determine the current in T; and B, a switch to make and break the current in M.

2. *Apparatus Employed in the Traction-Method.*

Fig. 2 shows front- and side-elevations of the apparatus employed in the method (presently to be described) of testing by traction. M is the magnet, with adjustable upper core C; K, the support, attached to the wall above; N, the weighing-device (comprising the spring Z, suspended from K by the wire W, and carrying the support O for the scale-pan O'; and also F, the graduated scale attached to K, and I, the indicator, attached to W); L, L' and L'', the wires used for adjusting the point of support of the scale-pan O', or the wire W, into the line of the axis of the core C (L and L' moving the wire W back and forth respectively in a line perpendicular to the wall X, while L'' moves it parallel to X); M', the switch-board; R-S, the reversing-switch; G, the galvanometer; T-C, the search-coil; S, the make-and-break switch; Q, the graduated scale, and A, the reel attached, through K, to the scale-pan O', and serving to measure the pull of the magnet upon the ore.

The magnet, specially made for these experiments, consisted (see M, in the side elevation) of a U-shaped mass of soft-iron, rectangular in section, through the extremities of which were openings, turned to fit large cylinders of similar material, which serve to complete the circuit. The lower cylinder (C′) was set firmly into the U-shaped frame, which was shrunk upon it; the upper cylinder was made to fit closely in the arm which supports it, yet sufficiently loose to allow adjustment, within a range which allows the ends of the cylinders to be brought together or separated ten inches.

Those portions of the cylindrical circuit enclosed between

FIG. 1.
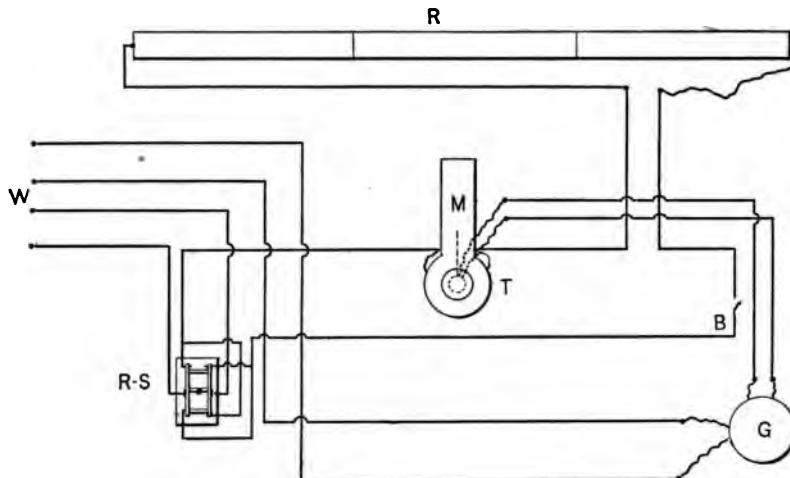
General Arrangement of the Magnetic Circuit.

the arms of the U-shaped frame were nearly as large in cross-section as the frame. The extremities of the arms were enlarged to receive the cylindrical circuit, the object being to maintain the same size of circuit throughout its entire length, and so not materially reduce the flux. Twelve inches of the cylindrical circuits of each arm were cut down to receive the exciting coils, thus forming the cores of the circuit. The coils were wedged fast to the cores, thus maintaining their relative position to the magnet, regardless of *its* position. The coils thus faced one another, enclosing the same axis, and, with the cores, could be brought together as the pole-pieces, giving a maximum concentration and a minimum leakage.

FIG. 2.
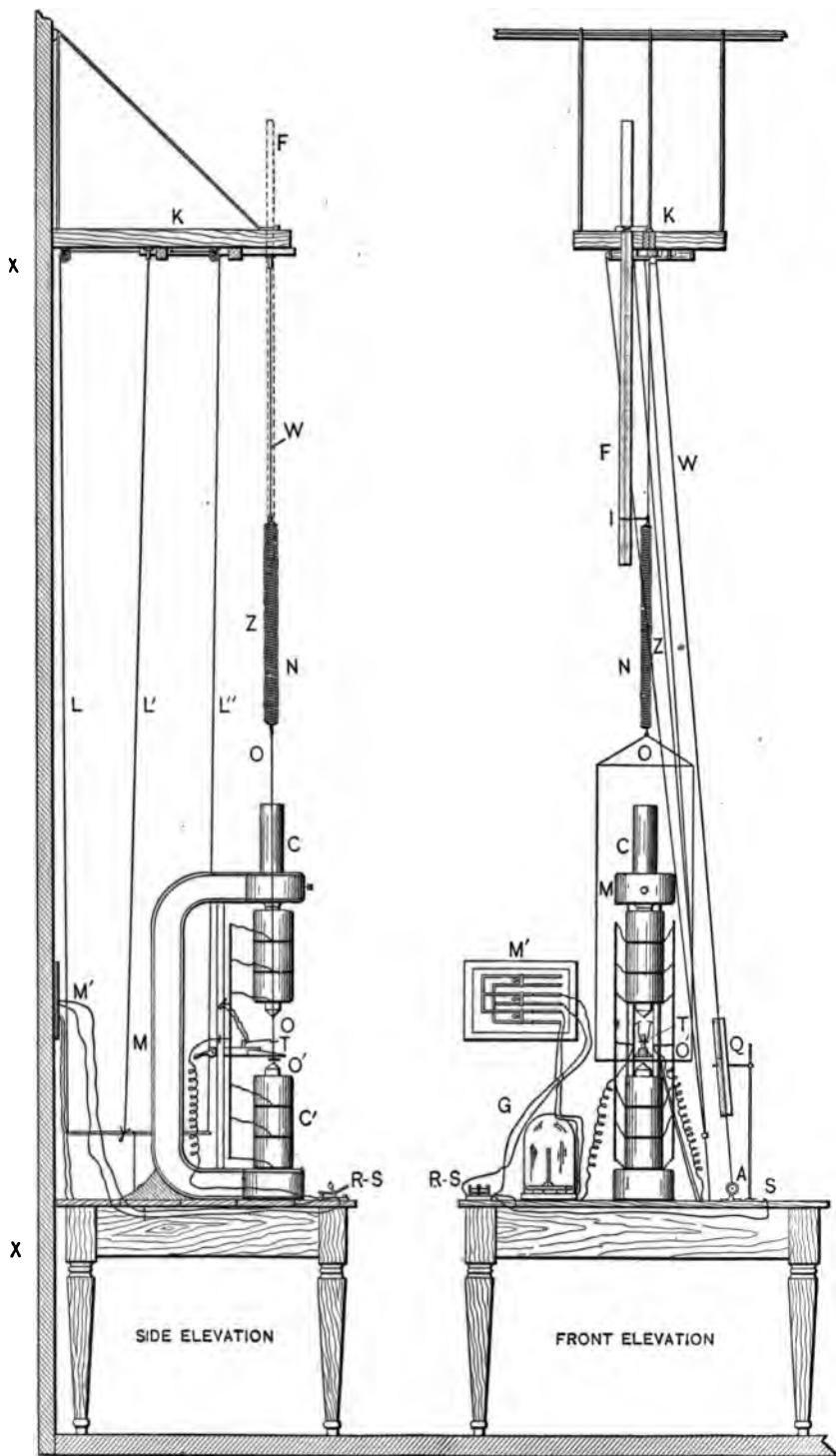
Apparatus Employed in the Traction-Method.

In the design of the magnet proper, the rules of the latest dynamo-construction were followed, except that the length of the circuit, which was necessitated by the required adjustment, is a radical departure from the latest design. The nature of the work required an adjustment of from 0.5 to 10 in.

a. The Coils.—As shown in Fig. 2, six coils were arranged in two groups—three on either pole. The coils were connected in multiple—primarily, to give maximum flux; secondarily, to facilitate the control of excitation by introducing or throwing out coils, as found desirable. The coils were of high resistance, allowing a current of only 3.5 ampères to pass on a 118-volt circuit. They were, however, wound to resist high temperatures, and would run continuously for several hours without overheating. Each coil was wound with 6500 turns of No. 24 B. & S. wire, giving, for the six coils, 39,000 turns, which, at 3.5 ampères, gives 136,500 ampère-turns of excitation.

b. The Pole-Pieces.—Tests were made with a large number of combinations of pole-pieces of different forms. Four sets of twos, with three odd ones, give twenty-odd possible combinations. The forms used were as follows: (1) Cones with rounded points, with semi-angle of 45°; (2) the same, with semi-angle of 52.5°; (3) cones with circular chisel-edges, the inner side being perpendicular and the outer side forming an angle of 60° with the base; (4) poles with a flat surface tangent to the outer curved edge; (5) poles with a spherical surface of 1.5 in. radius; (6) cones with a semi-angle of 60°, tipped with an inch-cylinder, which in turn is finished off with a spherical surface of 0.5 in. radius; (7) a concave, spherical surface of 4 in. radius, the inner surface being tangent to the outer curved edge.

3. *Methods of Testing Fields Produced by Different Pole-Pieces.*

The fields produced by differently formed pole-pieces of different forms were tested in two ways: first, by means of a search-coil; second, by a new application (as we believe) of an old principle, namely, the use of iron filings in a glass tube. By this method, both vertical and horizontal fields can be tested with equal facility, and both the arrangement and intensity of the lines of force in the field can be determined with great accuracy.

c. Testing with a Search-Coil.—The fields produced by all the forms of pole-pieces used were symmetrical with respect to the central axis of the cores. Tests made, therefore, from the center outward in a radial direction would be duplicated for any other radius which might be taken. The only possible point where the field might vary was on the side opposite the frame; but tests at that point did not show any appreciable change in intensity. We may therefore consider the field as uniform from the center outwards, radially.

The search-coil used in testing the fields was the same as that used in the tractive method, to be described later. It consisted of 100 turns of very fine, insulated copper wire, wound with a rectangular cross-section so closely as to occupy little space, and finally dipped in glue, to make it retain its shape, and also to protect it. The inside diameter was 0.5 in., the same as that of the pan in the tractive method, making the area enclosed by the coil 1.2664 sq. cm.

To facilitate the work of testing, the magnet was set so that the axis of the pole-pieces was vertical. The search-coil was first passed just above the top of the lower pole-piece, starting with the coil encircling the axis, then proceeding laterally, so that the search-coil should occupy, successively, positions distant by 0.5, 1, 1.5, 2, 2.5 and 3 inches from the starting-point. The coil was then passed through the field a second time, but this time equidistant from both poles, and tests were recorded at points located, as in the first series.

In Table I., giving the results of these tests (see p. 18), the first series is designated by A; the second by B. C is employed for a third series, made with the coil passing just below the upper pole-piece.

d. Testing with Iron-Filings and Glass Tube.—A glass tube 0.25 or 0.5 in. in diameter, containing a quantity of iron or steel filings (readily determined by experiment) proportional to the diameter of the tube, closed at the ends, and introduced into a magnetic field, is a useful means of determining the strength and direction of the field. The filings arrange themselves in small columns, which are close together or at some distance apart, according to the intensity of the field. To insure the perfect arrangement of the columns, the tube should be both turned and drawn back and forth in the field.

In measuring the intensity of the field the smaller tube does

fully as well as the larger, and has the additional advantage that it can be set in the groove of an architect's or draughts-man's scale, and, by slipping one upon the other, exact meas-urements of the field can be taken. The large tube serves best for determining the angle of divergence in weaker fields, the columns of filings rolling end-over-end in the tube, as it is moved across the diverging fields, as along the side of a magnet.

4. General Forms of Pole-Pieces.

The various forms of pole-pieces already mentioned may be classed as flat, conical, spherical (concave or convex) and chisel-shaped. These are the four most common forms; and the fields produced by them should be characteristic.

e. Flat Poles* (Fig. 31).—These pole-pieces were made flat for the full width of the core, the flat surface being tangent to the rounded edge. By rounding off the edge and making it tangent to the face, it was believed that the concentration of the lines of force would be rendered smaller than with square edge, thus producing a more uniform field on the face. This condition was realized to a certain extent; but on examining the fields produced by these flat poles used together, and also combined with different forms, marked variations in density of field will be seen to occur. The flat poles are useful, when combined with pointed poles, in producing fields of great diver-gence; but the divergence is rather slight at the point, for dis-tances varying from 2 to 6 and 8 in. On bringing the poles closer together, the divergence increases at the point; but the poles have to be brought closer together than is necessary with several other combinations to produce the same divergence.

f. Conical Poles† (Figs. 3 and 11).—Cones with semi-angles of 45° and 52½° were chosen to give maximum concentration, and are found, on trial, to give not only maximum concentra-tion, but maximum divergence from apex to cone. The cone with semi-angle of 45° gave maximum concentration; and poles of this form were used in the traction-method.

g. Spherical Poles (Figs. 19 and 28).—A concave pole-piece

---

* See Poggendorff's *Annalen*, vol. lxxiv., p. 465; vol. lxxx., p. 497 (1850); vol. xc., p. 248 (1853); vol. cv., p. 49 (1858). Also, *The Electromagnet and Electromag-netism*, by S. Thompson.

† J. A. Ewing, *Magnetic Induction in Iron and Other Metals*, New York, 1892, p. 141.

with 4 in. radius of curvature, and corners rounded and tangent to inner surface, was tested with all other forms. The result, in all cases, was a massing of the lines at the edges, leaving the concavity practically void of flux. The convex-spherical surfaces produce fields much resembling those of the conical forms, but exhibiting less concentration.

h. Chisel-Edged (Chamfered) Poles (Fig. 7).—Only one form of chisel-edge pole-piece was tested. It consisted of a cone of 60° semi-angle, with an inch hole drilled in the center. The chisel-edge was thus 60° on the outside and 90° on the inside, and of circular form, in the use of which, the field is symmetrical and suffers no distortion due to points and corners. When this form was combined with conical and spherical pole-pieces, strong fields were produced, decreasing from the center outward; when it was combined with flat and concave surfaces, the result was a varying field, which increased in intensity from the center to a line connecting the circular edges above and below, then fell off to a minimum in the outside field.

II. MAGNETIC FIELDS.

The fields produced by the different forms of pole-pieces described above may be classed as uniform, ellipsoidal, and conical or conoidal. Fields wholly or in part uniform are rare, even under the most favorable conditions. Fields No. 29 and 30, that portion of No. 30 included between the broken lines, that portion of No. 31 included between the broken lines, and a small portion of the middle part of the ellipsoidal fields (the width depending on pole-distance) give fairly uniform results.

Ellipsoidal fields may be divided into four classes, according to the shape of the pole-pieces forming them, as (1) those which are symmetrical with respect to a plane perpendicular to the axis at the middle point illustrated by Figs. 3, 11 and 16; (2) those which are unsymmetrical with respect to such a plane, one end of the field being wider than the other (illustrated by Figs. 8, 9, 10, 12, 13 and 14, of which 8, 9 and 10 might well be placed in the preceding class, as they show there is but slight difference between the unsymmetrical ends); (3) annular fields (of which there is but one example, namely, Fig. 7); (4) cupped fields (illustrated by Figs. 4, 5, 6 and 15), in which the whole field is strongly magnetic, but portions are so extremely dense as to give the appearance shown in the diagrams.

FIG. 3
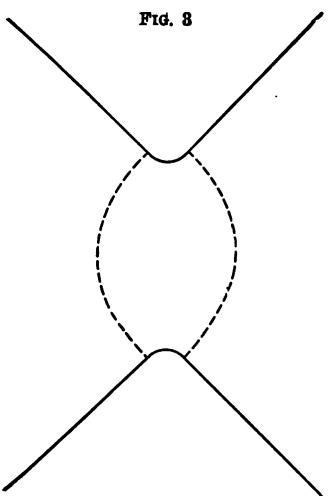
et of Cones with Rounded Points, Semi-
angle 45°. Pole-distance, 2 in.

FIG. 4
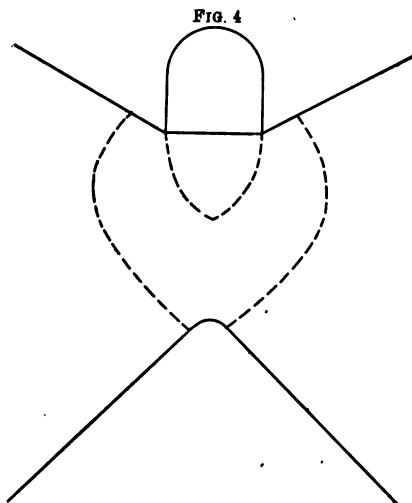
Combination of Circular Chisel-Edged
Poles and Conical Pole of Semi-angle
45°.   Pole-distance, 2 in.

FIG. 5
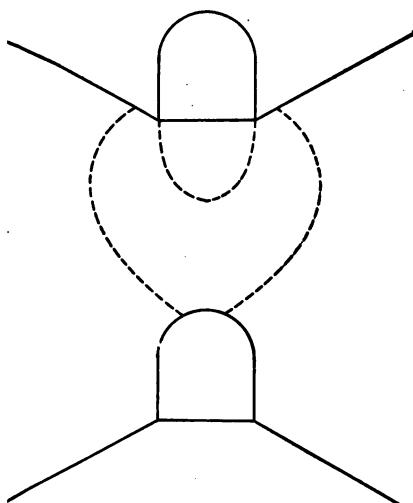
Combination of Circular Chisel-Edged
Pole and Spherical Pole of Half-inch
Radius.   Pole-distance, 2 in.

FIG. 6
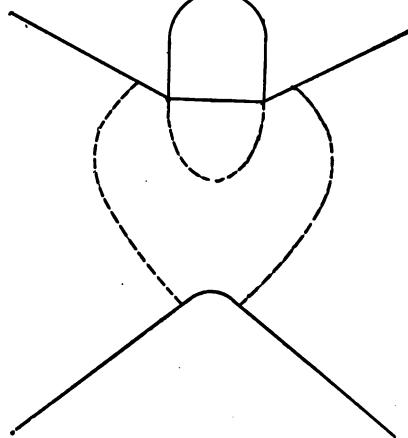
Combination of Circular Chisel-Edged
Pole and Conical Pole of Semi-angle
52.5°.   Pole-distance, 2 in.

FIG. 7
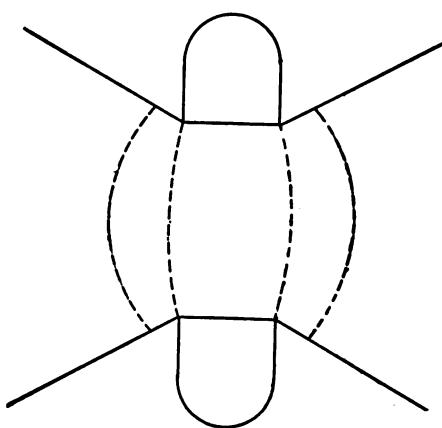
Set of Circular Chisel-Edged Poles.
Pole-distance, 2 in.

FIG. 8
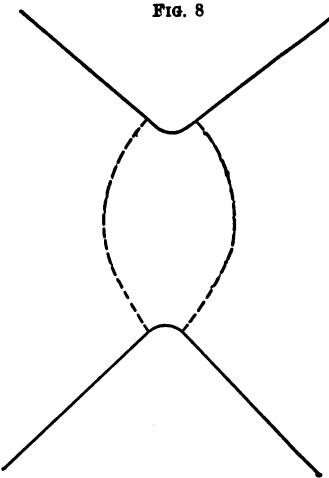
Combination of Two Conical Poles, of
Semi-angle 52.5° and 45° Respec-
tively. Pole-distance, 2 in.

FIG. 9
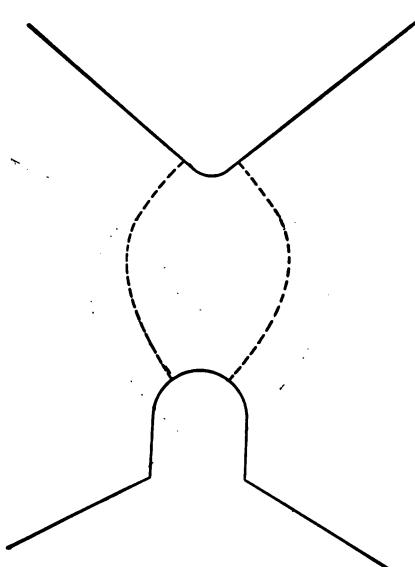
Combination of a Conical Pole of Semi-
angle 52.5° and a Half-inch Spherical
Pole. Pole-distance, 2 in.

FIG. 10
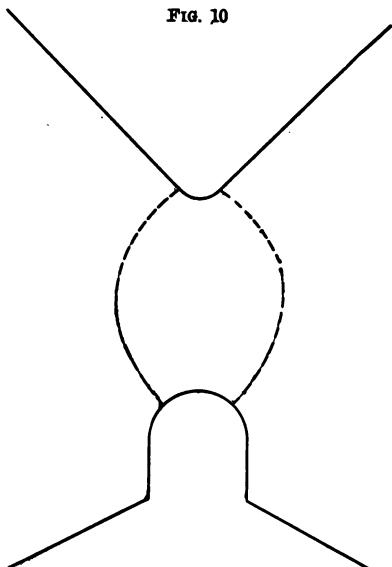
Combination of a Conical Pole of Semi-
angle 45° and a Half-inch Spherical
Pole. Pole-distance, 2 in.

FIG. 12

FIG. II
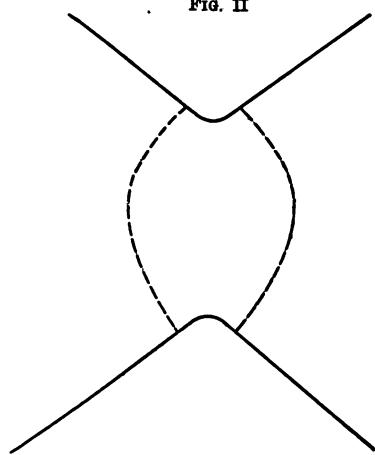
Set of Conical Poles with Rounded
Points of Semi-Angle of 52.5°. Pole-
distance, 2 in.
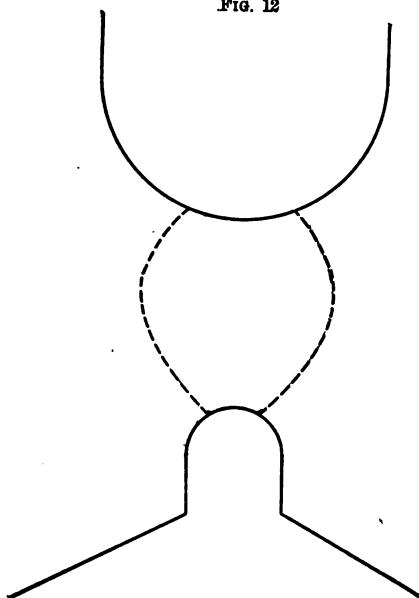
Combination of Two Spherical Poles
with Radius of 1.5 and 0.5-in. Re-
spectively. Pole-distance, 2 in.

FIG. 13
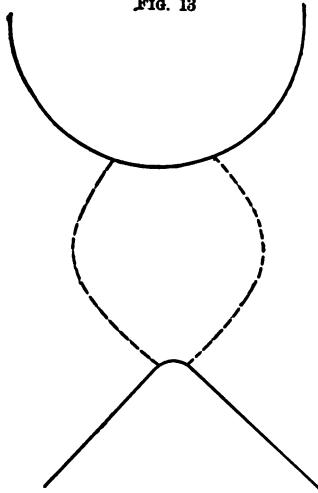
Combination of a Spherical Pole of 1.5-
in. Radius and a Conical Pole with
Rounded Point of Semi-Angle 45°.
Pole-distance, 2 in.

FIG. 14
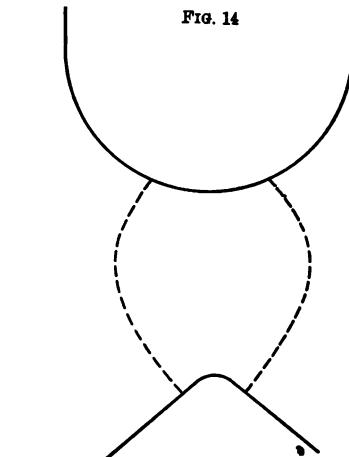
Combination of a Spherical Pole of 1.5-
in. Radius and a Conical Pole of Semi-
angle 52.5°. Pole-distance, 2 in.

FIG. 16

FIG. 15
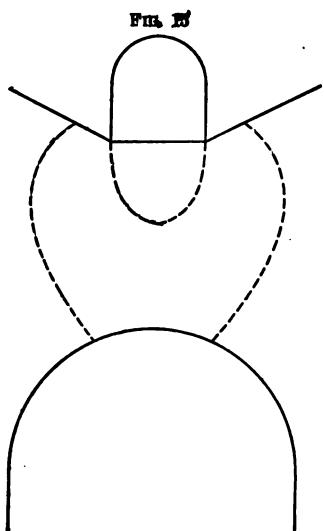
Combination of a Circular Chisel-Edged
 Pole and a Spherical Pole of 1.5-in.
 Radius.   Pole-distance, 2 in.
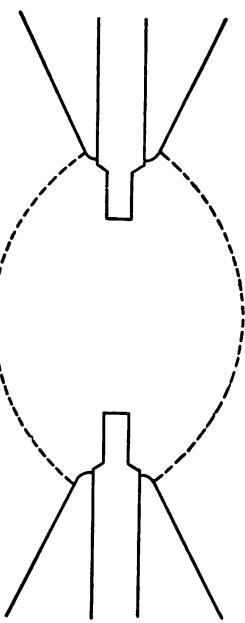
Set of Conical Pole-Pieces, Fitted with
 Adjustable Plugs, as Used in the Rod-
 Method of Testing.   Pole-distance,
 2 in.

FIG. 17
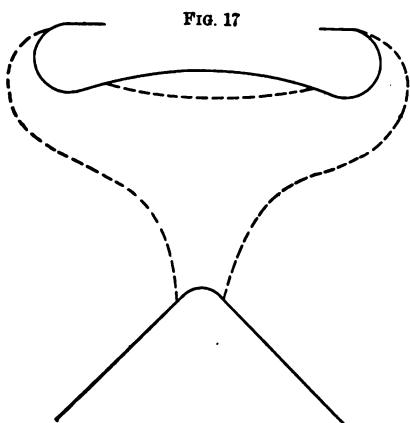
FIG. 18

Combination of a Concave Pole (4-in.
 Radius of Curvature) with a Conical
 Pole (Semi-angle 45°).   Pole-distance,
 2 in.
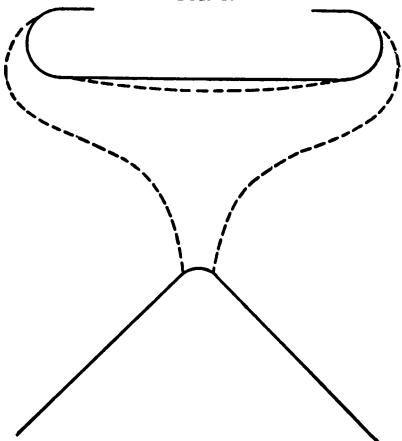
Combination of a Flat Pole (1.5-in.
 Radius) and a Conical Pole (Semi-
 angle 45°).   Pole-distance, 2 in.

FIG. 19
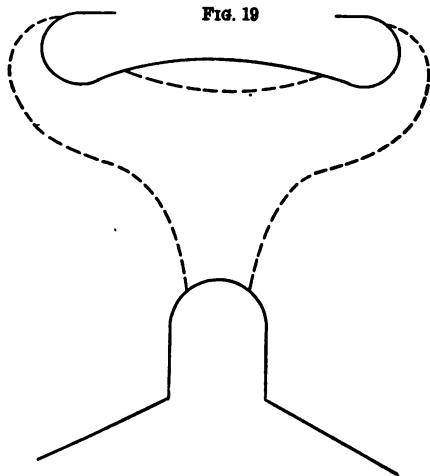
Combination of a Concave Pole-Piece (4-in. Radius of Curvature) and a Spherical Pole (1.5-in. Radius). Pole-distance, 2 in.

FIG. 20
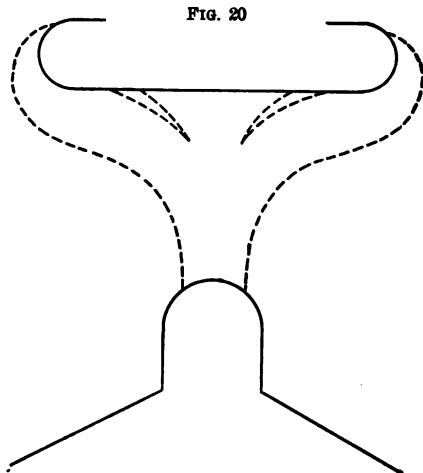
Combination of a Flat Pole (1.5-in. Radius) and a Spherical Pole (0.5-in. Radius). Pole-distance, 2 in.

FIG. 21
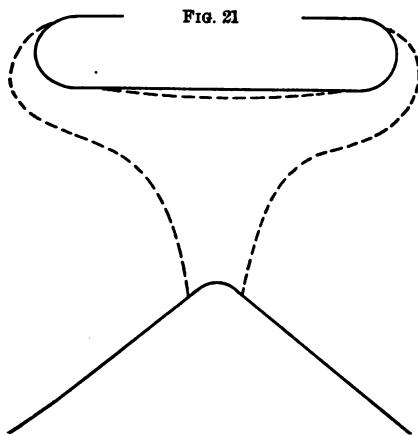
Combination of a Flat Pole (1.5-in. Radius) and a Conical Pole (Semi-angle 52.5°). Pole-distance, 2 in.

FIG. 22
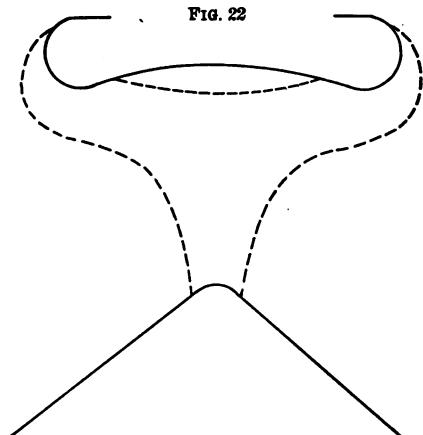
Combination of a Concave Pole-Piece (4-in. Radius of Curvature) and a Conical Pole (Semi-angle 52.5°). Pole-distance, 2 in.

Fig 23
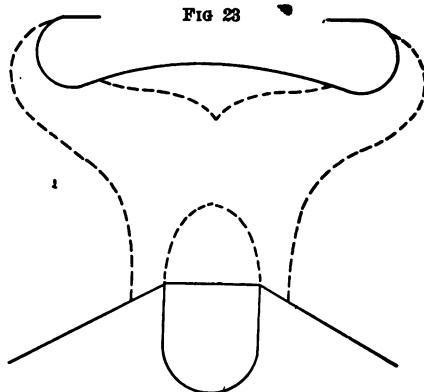
Combination of a Concave Pole-Piece
(4-in. Radius of Curvature) and a
Circular Chisel-Edged Pole.   Pole-
distance, 2 in.

Fig. 24
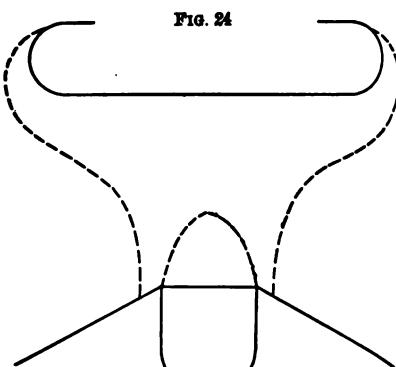
Combination of a Flat Pole-Piece (1.5-
in. Radius) and a Circular Chisel-
Edged Pole-Piece.   Pole-distance,
2 in.

Fig. 25
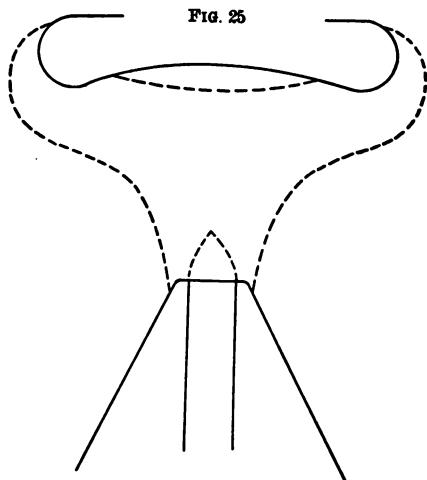
Combination of a Concave Pole-Piece
(4-in. Radius of Curvature) and a
Hollow Pole-Piece, as Used in the
Rod-Method.   Pole-distance, 2 in.

Fig. 26
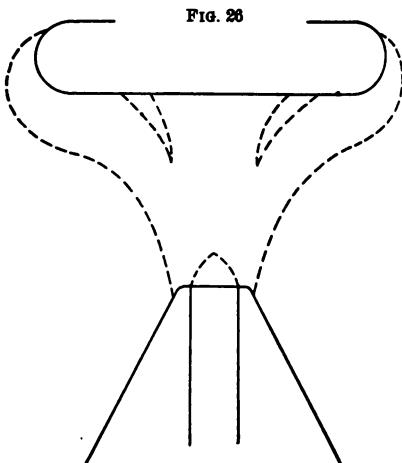
Combination of a Flat Pole-Piece (1.5-
in. Radius) and a Hole Pole, as Used
in the Rod-Method.   Pole-distance,
2 in.
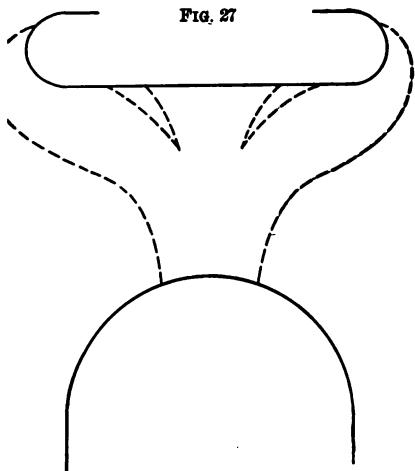
FIG. 27

Combination of a Flat Pole-Piece (1.5-
in. Radius) with a Spherical Pole
(1.5-in. Radius). Pole-distance, 2 in.
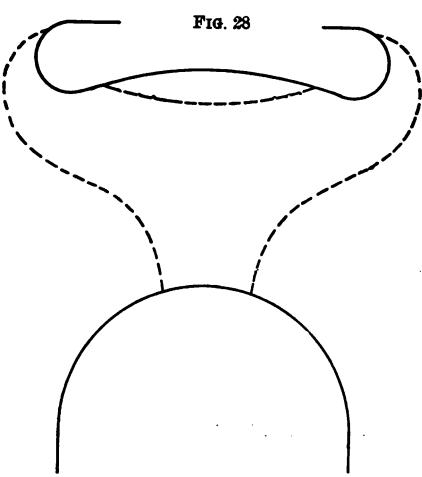
FIG. 28

Combination of a Concave Pole-Piece
(4-in Radius of Curvature) and a
Spherical Pole (1.5-in. Radius). Pole-
distance, 2 in.
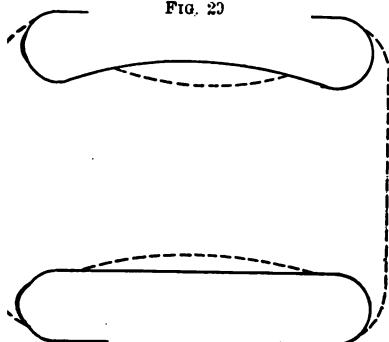
FIG. 30

FIG. 29

Combination of a Concave Pole-Piece
(4-in Radius of Curvature) and a Flat
Pole (1.5-in. Radius). Pole-distance,
2 in.
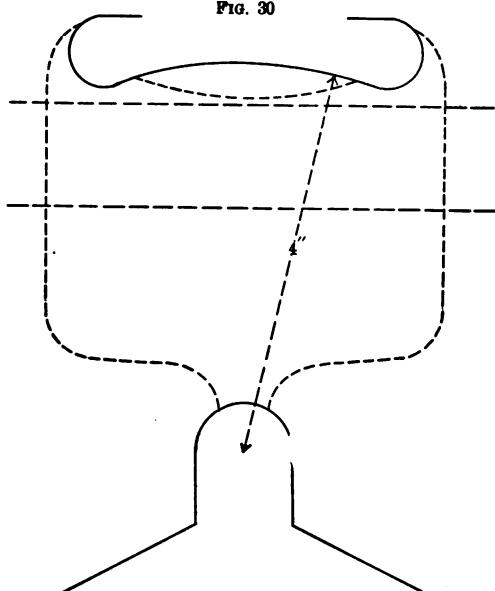
Combination of a Concave Pole-Piece
(4-in. Radius of Curvature) and a
Spherical Pole (1.5-in. Radius; the
Center of the Spherical Surface of the
Lower Pole being the Center of the
Curvature of the Concave Pole). Pole-
distance, 2 in.

FIG. 32

FIG. 31

Set of Flat Poles
(1.5-in. Radius).
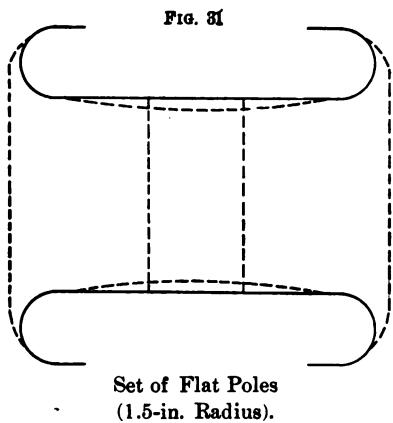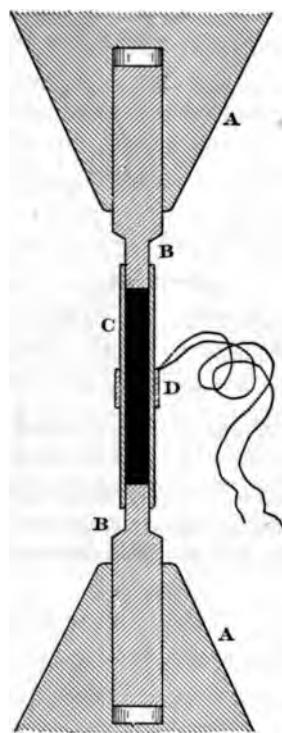
Apparatus for Glass-Tube Method.   A,
pole-pieces ; B, adjustable plugs ; C,
glass tube ; D, test-coil.

FIG. 33.

A

PAN. ACTUAL SIZE

B

PAN SUPPORT, ⅓ SIZE

Details of Pan and Support.
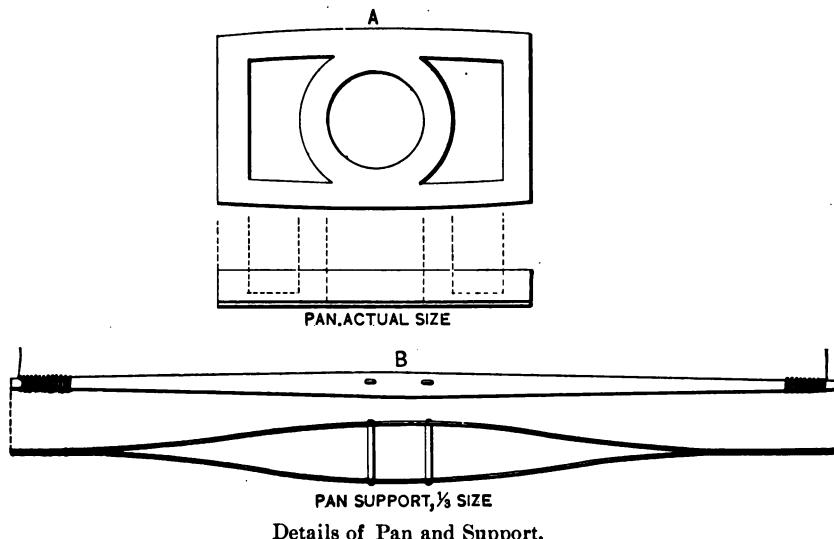
Conical fields are almost as rare as uniform fields. They merge into the conoidal shape so quickly that only small portions of any field can be said to be conical. However, the lower portions in Figs. 17, 18, 19, 20, 21, 22, 27 and 28 may be considered as conical.

Only one form of conoidal field was obtained, namely, the " ogee," which may be classed as cupped, solid, or ringed.

The cupped fields are due to projecting portions of pole-pieces, which tend to concentrate the lines of force. Here, as in the cupped ellipsoidal fields mentioned above, the great differences in density of field are accountable for the marked contrast between the different portions of the fields. Figs. 17, 18, 19, 21, 22, 23, 25, 28, 29, 30 and 31 are illustrations.

As to the solid ogee conoidal fields, it must be said that no arrangement of pole-pieces was found, producing conical and conoidal fields, which would give uniform results all the way across the upper pole-face, except that of Fig. 24, which gave a slight decrease from the center outward, but was cupped below. Figs. 18, 21 and 31 came nearest to being uniform, but, as the diagrams show, were less dense in the center.

In the ringed fields the central and outer portions are dense, while an annular space between is much less dense. This is best shown in Figs. 20, 26 and 27.

Table I. gives the results of search-coil tests on the fields described above. As already observed (see p. 6 above), there are two series of tests, here recorded as A and B; those of B having been made midway between the poles—the total pole-distance being the same for both A and B. The first column of the table refers to the figures which show the forms of the poles, etc., and in these figures the dotted lines give the results of the filing-tests. Table and diagrams, taken together, thus represent all the tests.

The combinations with pole-pieces shown in Figs. 16, 25 and 26, which were employed in the rod-method, and are described below under that title, were not intended to be used in the investigations on field-form and strength, but were tested nevertheless, and the results are given in the table.

5. Methods of Testing for Magnetic Permeability.

Two methods were employed in these tests: the rod-method and the traction-method.

TABLE I.—*Number of Lines of Force in Each 1.2664 Sq. Cm. of Field, at Successive Distances from the Center.*

Figs.		DISTANCES FROM THE CENTER, IN INCHES.						
		0.	0.5.	1.	1.5.	2.	2.5.	3.
3	A	8708.975	3751.559	2411.716	1751.794	1339.842	937.888	669.910
	B	2676.497	2411.716	1885.779	1504.623	1179.061	913.092	643.124
4	A	8708.975	4019.527	2465.310	1885.779	1339.842	913.092	643.124
	B	2576.497	2411.716	2050.560	1617.811	1179.061	913.092	643.124
5	A	9218.111	4702.651	2280.935	1617.811	1133.655	913.093	643.124
	B	2679.685	2411.716	2050.560	1347.029	1125.467	913.093	643.124
6	A	8574.992	4019.527	2679.685	2143.748	1504.623	1071.874	803.905
	B	3112.434	2679.685	2197.341	1779.592	1339.842	913.092	643.124
7	A	3215.662	4585.464	3483.590	2411.716	1504.623	1071.874	697.718
	B	2947.652	2947.652	2518.903	1939.373	1617.811	1071.874	697.718
8	A	8950.147	3590.777	2411.716	1724.998	1339.842	913.092	643.124
	B	2518.903	2411.716	2050.560	1347.029	1179.061	913.092	643.124
9	A	7664.899	4585.464	2143.748	1617.811	1179.061	913.092	643.124
	B	2518.903	2305.529	1939.373	1617.811	1179.061	913.092	643.124
10	A	8307.023	3858.746	1885.779	1347.029	1071.874	803.905	643.124
	B	2411.716	1885.779	1724.998	1339.842	1071.874	697.718	643.124
11	A	8092.648	4019.527	2518 903	1885.779	1339.842	913.092	643.124
	B	2518.903	2250.935	1885.779	1617.811	1339.842	913.092	643 124
12	A	8682.179	4126.714	2197.341	1617.811	1179.061	913.092	643.124
	B	2679.685	2250.935	1885.779	1347.029	1071.874	913.092	643.124
13	A	4126.714	3751.559	2411.716	1724.998	1339.842	913.092	643.124
	B	2679.685	2250.935	2050.560	1617.811	1071.874	803.905	535.937
14	A	7771.086	3858.746	2411.716	1885.779	1339.842	913.092	643.124
	B	2733.278	2465.310	1885.779	1347.029	1133.655	913 092	643.124
15	A	5466.564	4434.682	3112.434	1939.373	1179.061	913.092	643.124
	B	3001.247	2679.685	2250.935	1724.998	1339.842	913.092	643.124
16	A	10718.740*	2250.935	1347.029	968.686	803.905	643.124	535.937
	B	1347.029	1339.842	1179.061	913.092	913.092	589.530	589.530
17	A	7245.149	4126.714	2411.716	2143.748	1347.029	1071.874	803.905
	B	3215.622	2947.652	2679.685	2305.529	1724.995	1179.061	803.905
18	A	7503.118	4434.683	2786.872	2143.748	1617.811	1179.061	803.905
	B	3590.777	3483.590	2947.652	2518.903	1885.779	1339.842	913.092
19	A	8574 992	5627.345	2679.685	2050.560	1617.811	1179.061	803.905
	B	3483.590	3215.622	2733.278	2250.935	1779.592	1133.655	857.499
20	A	6270.462	4823.433	2576.497	1779.592	1339.842	1179.061	803.905
	B	2947.652	2786.872	2679.685	2250.935	1885.779	1179.061	803.905
21	A	7503.118	5091.401	3322.809	2411.716	2318.809	1724.998	1179.061
	B	3483.590	3322.809	2947.652	2518.903	2050.560	1347.029	1071.874
22	A	7084.368	4702.651	3215.622	2411.716	1671.404	1179.061	913.092
	B	3377.403	3215.622	2947.652	2679.685	2143.748	1347.029	1071.874
23	A	4327.496	4702.651	4126.714	3001.274	2050.560	1393.436	913.092
	B	3751.559	3590.777	3377.403	2841.466	2143.748	1393.436	913.092
24	A	4585.464	5680.939	4126.714	3215.622	2143.748	1347.029	1071.874
	B	8574.992	4126.714	3751.559	3112.434	2197.341	1617.811	1071.874
25	A	7610.305	6163.275	2054.840	2143.748	1617.811	1071.874	803.905
	B	3483.590	3322.809	3001 247	2518.903	1885.779	1339.842	913.095
26	A	7664.899	6538.431	2841.466	2197.341	1617.811	1179.061	1179.051
	B	3751.559	3483.590	2054.840	2518.903	1885.779	1179.061	803.905
27	A	6431.244	5466.564	4019.527	2518.903	1724.998	1179.061	913.092
	B	3858.746	3590.777	3215.622	2576.497	1885.779	1347.029	913.092
28	A	6699.252	5895.307	4019.527	2679.685	1724.998	1133.655	913.092
	B	3751.559	3590.777	3215.622	2679.685	2143.748	1724.998	913.092

* Plug tip enclosed by coil.

TABLE I.—*Continued.*

Figs.		DISTANCES FROM THE CENTER, IN INCHES.						
		0.	0.5.	1.	1.5.	2.	2.5.	3.
29	A	4019.527	4019.527	4327.496	4702.651	2143.748	1179.061	803.905
	B........	3537.184	3215.622	3590.777	3215.622	2518.903	1939.373	1347.029
30	A	6977.181	3590.777	1939.373	1617.811	913.092	535.937	375.155
	B........	1885.779	1724.998	1671.404	1347.029	1071.874	803.905	535.937
	C*......	1939.373	1939.373	2250.935	3322.809	1885.779	913.902	697.718
31	A	4019.527	4019.527	4126.714	5091.401	2411.716	1339.842	913.092
	B........	3751.559	3751.559	3590.677	3112.434	2143.748	1347.029	913.092

i. The Rod-Method.—This is a modification of a method frequently used in the laboratory to determine the magnetic permeability of iron and steel rods. As described by Ewing,† it consists in placing an exciting coil and a search-coil, side by side, on a long rod of the material to be tested. By suddenly jerking off the search-coil, and removing it to a distance such that the induction is reduced to zero, the flux passing through the rod can be determined. The quantity B (the degree of magnetization in the material) could thus be determined. By repeating the operation as before, with the exception that the test-rod is replaced by a glass tube or wooden rod, H (the degree of magnetization in air) can be determined. The ratio of the flux in the rod to the flux in air will give the permeability μ of the rod.

This method, however, is not exact, for, first, the rod would have to be very long to insure a uniform field in which to place the search-coil; secondly, the replacing of the test-rod by glass or wood would so affect the field that H could not be determined with accuracy; and thirdly, the resistance of the return-circuit for the flux would be infinitely great, whereas it should be infinitely small, in order that as many lines of force as possible would be forced through the test-piece.

Another difficulty, preventing the application of this method, was that long rods of metal, and especially of pure minerals, could not be obtained.

* A third series of tests, made by passing the coil just below the upper pole-piece.

† *Magnetic Induction in Iron and Other Metals,* p. 65.

The following method, which somewhat resembles Ewing's Isthmus Method,* was therefore adopted.

j. Method Employed in These Experiments.—Pole-pieces, conical in form, with semi-angle of approximately 45°, were turned to fit the cores of the magnet already described. A half-inch hole was drilled and reamed through the central part of the cone nearly to the base. Soft iron plugs were turned to fit the holes, and of a size to allow adjustment. One end of each plug was turned down for a distance of 0.5 in. to 0.25 in. diameter. The exact form and dimensions of these pole-pieces and plugs can be seen in Fig. 32.

A glass tube of 0.25 in. inside diameter, encircled at the middle by a search-coil, could be placed between the two adjustable pole-pieces, which were separated by several inches, and the smaller ends of the same could be inserted in the ends of the glass tube. The tube was graduated and marked at two extreme points (2 in. apart), which served to indicate the amount of material as compared with the standard 2-in. metal rods tested.

Search-coils of two sizes, one having 100 and the other 500 turns of wire of the same size, were used: the coil of fewest turns in testing the more highly magnetic, and the other for the more feebly magnetic substances. The deflection of the galvanometer is increased and the possibility of errors in reading is reduced by the use of the larger coil.

The material to be tested is placed in the glass tube. From time to time, while the tube is being filled, the pole-pieces (the plugs) are introduced, and the current is turned on. In this way the material is compressed until when it reaches the upper mark no further compression is possible with the plugs under the force acting. With rods this is, of course, necessary. When properly filled, the ends of the plugs are adjusted to occupy positions equidistant from the apexes of the conical pole-pieces. The coil is also adjusted so as to be midway of the material to be tested in the tube.

By reversing the current, the flux through the magnet is re-

---

* " The Isthmus Method." *Magnetic Induction in Iron and Other Metals,* by J. A. Ewing, p. 132. "On the Magnetization of Iron and Other Metals in Very Strong Fields," *Philos. Trans. Roy. Soc. of London,* Vol. clxxx., J. A. Ewing and Wm. Low.

duced from a maximum to zero.   One-half of the throw is then taken as a measure of the flux.*

B can be determined, and H also, by introducing a small piece of wood to keep the plugs at the proper distance apart.

This method has all the advantages of the rod-method, and none of its disadvantages.   If solid rods of pure mineral could be obtained, they could be tested in the same manner.   But such rods can be obtained for a few minerals only, and for this reason, as well as to save time and cost, crushed material was used in the tests.

k. Difference in the Results of the Two Methods.—By reason of the large proportion of air-space existing between the particles, when crushed material is tested, the quantity obtained from the ratio $\frac{B}{H}$ is not the true permeability.   In cast-iron, for example, the crushed product, unsized below 190-mesh, gives only 33.79 per cent. of the permeability obtained for a rod of the same material.   In other words, there is a fall of 66.21 per cent. from the actual permeability, as appears from the following tests made on a number of metallic iron compounds :

TABLE II.—*Comparative Tests of Filings and Solid Rods.*

	Value of μ.	Percentage of Rod-Test.
1. Wrought iron, rod, ...	7.177	
Wrought iron, filings, ...	2.387	33.25
2. Steel casting, rod, ...	7.016	
Steel casting, filings, ...	2.419	34.47
3. Tool steel, rod, ...	6.612	
Tool steel, filings, ...	2.338	35.36
4. Cast-iron, rod, ...	6.120	
Cast-iron, filings, ...	1.889	30.80
Average percentage of rod-test, ...		33.47

The attempt to find whether this variation would be the same for other metals failed, because the method was not delicate enough.   The following metals were tested in comparison with filings of them without noting any difference of flux.   The flux was in all cases the same as that of air, namely, aluminum, carbon, cadmium, copper, lead, magnesium, tin and zinc,

---

* This was possible, by reason of the softness of the metal composing the metallic circuit.

when tested in rods, showed a permeability of 1.   Of the other minerals tested in the form of powder, only three gave any results, namely, franklinite (1.482), ilmenite (1.241) and pyrrhotite (1.068).

Rods of these minerals not being obtainable for testing, no data as to their permeability in that form could be secured. We do not know, therefore, whether there is any change in the permeability as a result of the reduction of size.   It is, however, permissible to assume that there is probably a falling-off, due to the introduction of air into the mass, especially for highly magnetic substances; but that for the less magnetic substances, which differ but little from air, the fall in permeability would be slight.   Of those having less permeability than air, the permeability may rise to nearly that of air.   How great the variation is it is hard to say; hence no correction of observations can be made, and only comparative results can be obtained, with crushed cast-iron as a basis.

The comparison of ores with cast-iron is based primarily on the fact that cast-iron can be crushed and sized like minerals. Yet such a comparison is faulty because, cast-iron being tougher than most ores, the grains produced by crushing it are more rounded in form than the irregular grains of ores reduced by the same process.   Filings can scarcely be used for comparison, being still more angular in form, or shaped like shavings.

In calculating B and H, a correction* must be made for the flux in the glass tube, and also in the coil itself.   If the search-coils be wound close and the insulation be very thin, with coils of fine wire and of few turns, this correction is practically negligible.   This would be the case with the 100-turn coil, as regards the flux included between the inside and outside; but for the 500-turn coil a correction must be made.   The correction for induction through the glass tube is necessary in both cases. These corrections can be made by employing the following formulas:

(1) $$B = \frac{F - (A - A')H}{A};$$

F being the total flux enclosed by the coil; A, the cross-section of material (test-sample or air) treated; and A' the cross-sectional area of glass tube and coil.

---

* J. A. Ewing, *Magnetic Induction in Iron and Other Metals*, p. 66.

$$(2) \qquad\qquad H = \frac{F}{A - A'}$$

when the glass tube has the same resistance as air, which is true in this case.

l. The Traction Method.—In the arrangement of the apparatus for this method, the large magnet is set upright on one end as a base; the end with the loose core being uppermost, to allow adjustment (see Fig. 2).

About 6 ft. directly above the magnet was a triangular frame of wood attached to the wall by its base and rods, and provided with a system of levers, operated by cords reaching to the table and within easy reach of the operator, by means of which a pulley on one of the levers could be brought into the axis of the cores. To one end of a small flexible copper wire, passing over this pulley, were attached a pan, spring, etc., to be described later. The other end extended to the table at the right of the magnet's base, and there engaged with the axle of a small windlass. The bearing at this end was formed of curved brass springs, so arranged as to press upon the axle and present considerable frictional resistance to its revolution. A disk at the other end of the axle furnished a means of turning the windlass.

The other end of the wire, hanging over the center of the upper core, was provided with a swivel, to which was fastened a wire, extending to the left, at right-angles to the pendant wire, and ending in a rectangular loop which enclosed a porcelain scale. This loop both prevented the twisting of the wire and served as an indicator.

To the swivel was attached a spring, and to the lower end of the spring a pan, in which was placed the ore, the attraction of which was to be measured.

The four springs were wrapped on a lathe to insure regular spirals, and were made respectively of Nos. 14, 15, 19 and 20 standard-gauge piano-wire. They are designated respectively by the letters A, B, C and D, when referred to in this paper.

Only one of these springs, D, could be calibrated. The others, by reason of slight irregularities in the spirals, refused to give uniform results. In all cases, therefore, the stretch in centimeters was noted, and its equivalent in grammes was found by suspending the spring and attaching a scale-pan, into which

weights were placed until the same stretch was obtained as had been noted in the experiment. This weight, plus the weight of scale-pan and attachment, gave the pull in grammes corresponding to the observed stretch in centimeters.

The calibration for spring D was found to be one gramme for every 0.55 cm. of stretch.

The pan in which the ore was placed* consisted of a piece of vulcanized fiber 0.13 in. in thickness, through which a half-inch hole had been drilled and reamed to exact size. To the bottom of this perforated piece was glued another thin piece of fiber, and the whole was cut to the desired form (Fig. 33). A large part of the fiber was cut away from the upper surface of the pan to give lightness, and to reduce the influence of the mass on the magnetic field and on the materials treated.

Two strips of seasoned hickory $\frac{3}{32}$-in. thick, $\frac{3}{8}$-in. wide at the middle and $\frac{2}{16}$-in. at the ends, and 3 in. long, curved at the edges to resist the strains produced by upward pull, were placed side by side, and the ends were wrapped with linen thread, the edges being slightly notched to give the wrapping a firmer hold. When securely fastened at the ends, the two strips were separated at the middle, and small triangular cross-pieces of similar material were mortised into the side strips—a flat side up. These cross-pieces were placed an eighth of an inch below the edge of the strips and an inch apart, thus forming a support for the pan. The pan proper had its sides cut to fit in between these side-strips, and, being made a little large, was firmly held when pressed into its seat.

The pan-support was suspended by a small copper wire attached to each end. These suspension-wires were kept parallel for a distance of some three feet by a round cross-bar of hickory, tapering in diameter from $\frac{1}{8}$-in. at the middle to $\frac{3}{32}$-in. at the ends. Above this cross-bar the two wires were united, and attached to the lower end of the detaching-spring described above.

By this arrangement the detaching-spring, and therefore the point at which the power of detaching was applied, were brought directly above the pan—pan and spring lying within the axis of the cores. By adjusting the point of support of the pan, the

---

* *Poggendorff's Annalen* (Plücker), vol. lxxiv., 1848, p. 321.

pan itself can be made to assume any position perpendicular to the axis of the cores. The arrangement of levers on the frame above, as described, facilitated this adjustment.

The scale, of which the position has already been indicated, was used in measuring the pull of highly magnetic substances, such as iron, steel, etc. Another scale was placed parallel with and very close to the wire leading from the windlass, and supported in place by a standard and clamp. A small, fine wire, attached at right-angles to the windlass wire, moved over the scale as the windlass was turned, thus serving as an indicator.

A wooden standard, wedged between the upper and lower arms and parallel to the bottom or back of the magnet, served as a support for feed-wires, leading to the galvanometer and the search-coil, and also for an adjustable fork-arrangement used in checking the upper movement of the pan-support. The whole apparatus as described, with minor details, is shown in Fig. 2, which will serve to elucidate the above description.

Frames supporting several thousand feet of galvanized-iron wire, in heliacal form, served as resistance.

A double reversing-switch, placed in direct connection with the main circuit, allowed a reversal of the current in the coils, and thus of the magnetization of the magnet. A small make-and-break switch was placed in line with one of the wires leading to the magnet from the reversing-switch, so as to allow the current to be thrown on or off at will without reversing.

A separate circuit connected the galvanometer with the mains, to secure a steady current through the former, which was of the direct-reading d'Arsonval type.

The method of operation was as follows: Conical pole-pieces, with semi-angle of 45°, were employed: first, to obtain maximum concentration; second, for convenience of use with the pan. When the suspension has been so adjusted as to bring the pan directly over the apex of the lower pole-piece, the pan is removed from the holder; the material to be tested is put in, settled, slightly compressed with the rounded end of a test-tube, and then struck off even, and rolled smooth with the side of the test-tube, all particles adhering to the pan being carefully removed. The pan is then placed in its support, which is then raised $\frac{1}{8}$-in. or more above the apex of the cone. When the oscillations of the pan have ceased, it is slowly lowered by means

of the windlass until its bottom just touches the pole-piece, as is shown by the cessation of the slight tremors due to the spring-attachment above.

The current is now turned on, and the full strength of the magnetic field is concentrated upon the material in the pan. Gravity being eliminated, the force necessary to detach the pan will be proportional to the divergence of the lines of force, the flux, susceptibility and mass of material treated, as will be explained further on.

By slowly turning the windlass, the spring is stretched until it overcomes the magnet's attraction; then the pan, with its support, springs upward and is caught on the two prongs of the fork, which extend one on either side of the field. By adjusting the fork so that it is only a fraction of an inch above the pan-support, when the pan is resting on the pole, the material in the pan is but slightly disturbed, or, at least, not spilt—so that the test can be repeated over and over again. It is necessary, however, to replace the contents of the pan, if disturbed in the least, for reasons which will be explained later.

By means of the calibrated spring, D, the centimeters of pull can be at once determined as grammes; but, with the other springs, each measurement of pull will have to be transformed, as previously described.

m. Effect of Loosening the Charge.—Several tests were made to determine the effect of loosening the contents of the pan. Consecutive tests, made without replacing and settling the charge, as in the initial test, gave the results shown in the following table:

TABLE III.—*Consecutive Tests, Without Replacement of Charge, Showing Pull in Centimeters of Scale.*

No.					Iron.	Steel.	Franklinite.	Ilmenite.
1,	.	.	.	.	1018	1000	325	175
2,	.	.	.	.	925	982	307	170
3,	.	.	.	.	885	948	305	180
4,	.	.	.	.	871	885	300	...
5,	.	.	.	.	880	837	...	...
6,	.	.	.	.	...	947	...	...

As will be seen from this Table, and the curves in Fig. 34, there is more or less sudden falling-off of the tractive force, but a final rise, most marked in the case of steel. Ilmenite is,
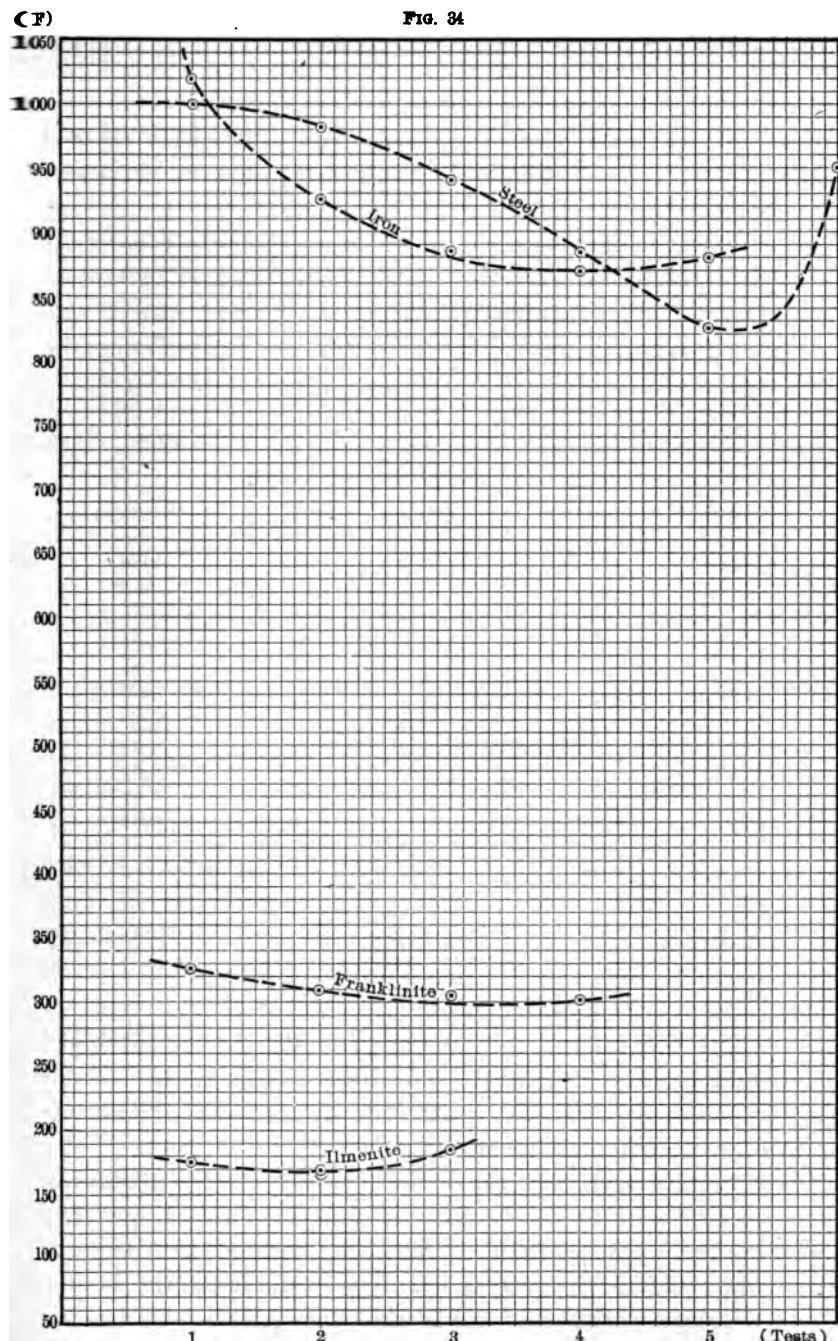
Curves Showing Variation in Tractive Force (F), Due to the Loosening of the Contents of the Pan, by Successive Tests without Intermediate Replacement. (See Table III.)

however, an exception, a slight rise being noted, with little or no fall.

The above results are the average of a number of tests, some of which gave little variation, others quite marked rises and falls, but all of which followed, in a general way, the variations as indicated above.

From the above it will be seen to be necessary to repack the charge carefully for each test. By so doing, and by careful manipulation of apparatus, results can be made to check with surprising accuracy, as is shown by two series of tests, given below:

TABLE IV.—*Tests Made with Repacking of Charge after Each, Showing Pull in Centimeters of Scale.*

Test No.								Quartz.	Hematite.
1,	.	.	.	.	.	.	.	600	1.450
2,	.	.	.	.	.	.	.	600	1.451
3,	.	.	.	.	.	.	.	601	1.450
4,	.	.	.	.	.	.	.	600	1.450
5,	.	.	.	.	.	.	.	600	1.452
6,	.	.	.	.	.	.	.	601	1.449
7,	.	.	.	.	.	.	.	601	1.450
8,	.	.	.	.	.	.	.	599	1.450
9,	.	.	.	.	.	.	.	602	1.451
10,	.	.	.	.	.	.	.	600	1.450
		Average,	.	.	.	.		6004	1.4503

The pan itself was found to be slightly magnetic, but this, being a constant element, introduced no error. It may even be considered as a positive advantage; for with the more feebly paramagnetic and the diamagnetic substances a pull could always be obtained, although often less than that of the empty pan. The difference between the pan-pull and the combined pan-and-mineral pull gave the pull due to the mineral. If this had not been the case, the method would not have been applicable with diamagnetic substances.

n. Corrections.—In the case of the highly magnetic substances, a correction must be made for the stretch of the suspending wires. This was partially obviated by taking the readings from the scale above the magnet. Here the indicator is connected directly with the end of the spring, thus eliminating the stretch of the wire from windlass to spring. There remains to be corrected the stretch of the suspension-wires. No

constant could be determined which might be subtracted for every gramme of pull; but the stretch for each test was determined by noting the change of starting-point—the difference between the two points giving the stretch of wire.

Another correction must be made for the fall of magnetization due to the heating of both coils and core. A series of tests will show this to the best advantage.

TABLE V.—*Showing Decrease of Magnetization Due to the Heating of Coils and Core.*

No. of Test.						Temperature. Deg. Fahr.	Time. Minutes.	Magnetization. (Lines of Force.)
1,	.	.	.	.	.	. 73.	0	11254.677
2,	.	.	.	.	.	. 74.8	10	11093.895
3,	.	.	.	.	.	. 78.25	20	10880.521
4,	.	.	.	.	.	. 82.65	30	10718.740
5,	.	.	.	.	.	. 87.75	40	7935.867
6,	.	.	.	.	.	. 93.40	50	7771.086

The effect of the heating is to increase the resistance of the coils and the reluctance of the magnetic circuit, in both ways reducing the magnetization. Two methods may be employed to obviate this decrease of magnetization, namely, increase the current proportionally or keep the coils and core at constant temperature. As the current was on during a small part only of the time of testing, a constant temperature could easily be maintained. A certain temperature was obtained by allowing the current to pass through the coils for several minutes, and was then kept constant. No correction is necessary, therefore, except with bismuth, which was tested with the magnet cold, which will account for the higher flux noted for the same pole-distance.

6. *Theory.*

The tractive force having been determined, the question arises, How can the permeability be obtained from it? Before this can be done, the quantities which combine to produce the tractive force must be determined. Tests were, therefore, made on : (a) varying weights of mineral, with equal pole-distance, and magnetization; (b) varying pole-distances, with equal volumes and magnetization; (c) varying magnetization, with equal volumes and pole-distances; and (d) varying size, all other conditions being constant.

o. Effect of Varying Weight on Tractive Force.—It was found necessary to employ equal volumes, filling the scale-pan each time, to insure equal divergence of the lines of force through the material treated. If equal weights had been taken, the experiments would have been made under different conditions in this respect. With equal weights, and therefore varying volumes, the ratio of air-gaps to the depth of material tested will vary, thus causing a variation in intensity and divergence of field. Again, with equal volumes of mineral, the conditions under which the grains act upon each other and upon the magnetic field will be the same in the different experiments.

The effect of varying weight will be seen in the following table:

TABLE VI.—*Variation in Tractive Force Due to Varying Weight of Mineral, as Shown by Tests Made on Hematite.*

Pole-Distance. Inches.				Weight of Mineral. Grammes.	Pull. Grammes.	Flux. 1.2668 Sq. Cm.	Lines per Sq. Cm.	Difference.
8,	.	.	.	. 1.38	0.909		2679.685	......
8,	.	.	.	. 1.19	0.836		"	0.073
8,	.	.	.	. 1.03	0.745		"	0.091
8,	.	.	.	. 0.85	0.709		"	0.036
8,	.	.	.	. 0.69	0.636		"	0.073
8,	.	.	.	. 0.51	0.527		"	0.109
8,	.	.	.	. 0.34	0.400		"	0.127
8,	.	.	.	. 0.17	0.154		"	0.246

On reference to Fig. 35, it will be seen that on starting with a certain amount of mineral and decreasing the weight by constantly diminishing fractions of the whole, we get a decrease in the tractive force. From the eight-eighths point (which represents the full amount taken), down to the six-eighths point, there is a gradual decrease; but from the latter point down the decrease becomes irregular, by reason both of a decrease of mass and an increase of divergence of lines of force. The pan being only partly filled, the center of gravity of the mass tested was nearer the pole-piece, in the tests of the smaller quantities. Connecting the points would give very nearly a continuous curve; but a straight line, *o a*, would probably represent the truth more nearly, as eliminating the effect of this lowering of the center of gravity.

We may therefore say that the tractive force varies directly with the mass or weight.

FIG. 35
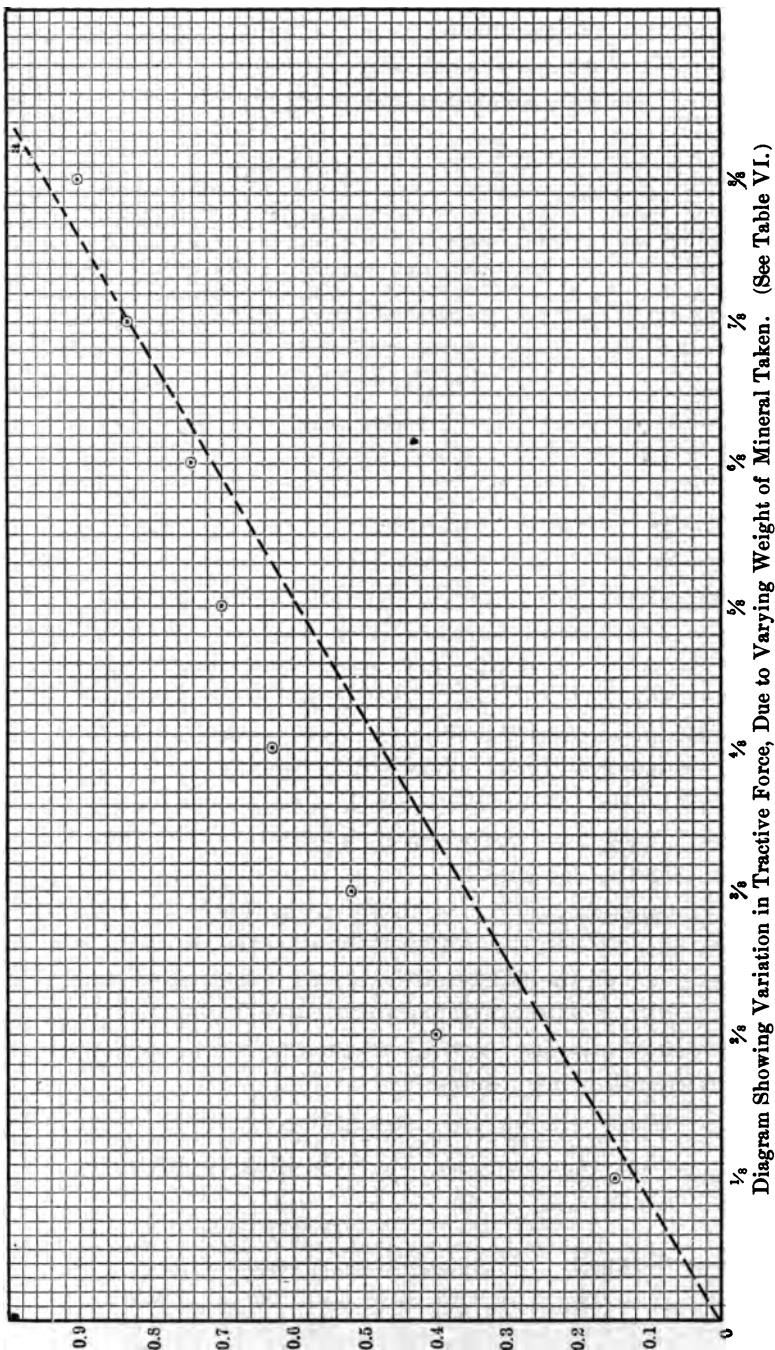
Diagram Showing Variation in Tractive Force, Due to Varying Weight of Mineral Taken. (See Table VI.)

p. Effect of Varying Pole-Distance.—Tests were made with equal volumes and masses of mineral, and varying pole-distances, but with a constant flux, maintained by increasing the current as the pole-distance was changed. The results were:

TABLE VII.— *Variations in Tractive Force Due to Varying Pole-Distance, as Shown by Tests Made on Pyrrhotite.*

Pole-Distance. Inches.				Pull. Grammes.	Pull. Centimeters.	Flux. Lines of Force per 1.2668 Sq. Cm.	Difference.
1,	.	.	.	. 3.181	1.750	2679.685	......
2,	.	.	.	. 3.363	1.850	"	0.182
3,	.	.	.	. 3.454	1.900	"	0.091
4,	.	.	.	. 3.727	2.050	"	0.273
5,	.	.	.	. 3.636	1.950	"	0.091
6,	.	.	.	. 3.454	1.900	"	0.182
7,	.	.	.	. 3.363	1.850	"	0.091
8,	.	.	.	. 3.000	1.650	"	0.363

In Fig. 36 it will be seen that for the first four inches the rise is fairly regular, but beyond the fourth inch there is a sudden, irregular fall, evidently due to the decreasing divergence of the lines of force. When the poles are close together, the divergence is slight, increases to a maximum at some point between 4 and 5 in. pole-distance, and decreases as the poles are further separated.   This is shown graphically in Fig. 38.

To what extent the outside field influences the distribution and action of the enclosed portion cannot be said at present. That it does have some effect is certain.   The line A B will represent the variation.

We conclude, therefore, that, within the limits indicated in Figs. 36 and 38, the tractive force varies directly with the divergence of the field, and hence with the pole-distance.

q. Effect of Varying Magnetization.—With equal volumes and pole-distances, but varying magnetization, a series of tests gave the following results:

TABLE VIII.— *Variations in Tractive Force Due to Varying Magnetization, as Shown by Tests Made on Hematite.*

Pole-Distance. Inches.	Pull. Grammes.	Flux. Lines of Force per 1.2668 Sq. Cm.	Difference.
3.218	0.636	1617.811	....
3.218	0.972	3215.622	0.3364
3.218	1.717	4823.433	0.7445
3.218	2.336	6431.244	0.6191
3.218	3.109	8039.055	0.7721

In Fig. 37 it will be seen that the plotting is quite regular, and a straight line will represent very well the variation of

FIG. 38

Diagram Showing Effect on Tractive Force by Varying Pole-Distance. (See Table VII.)
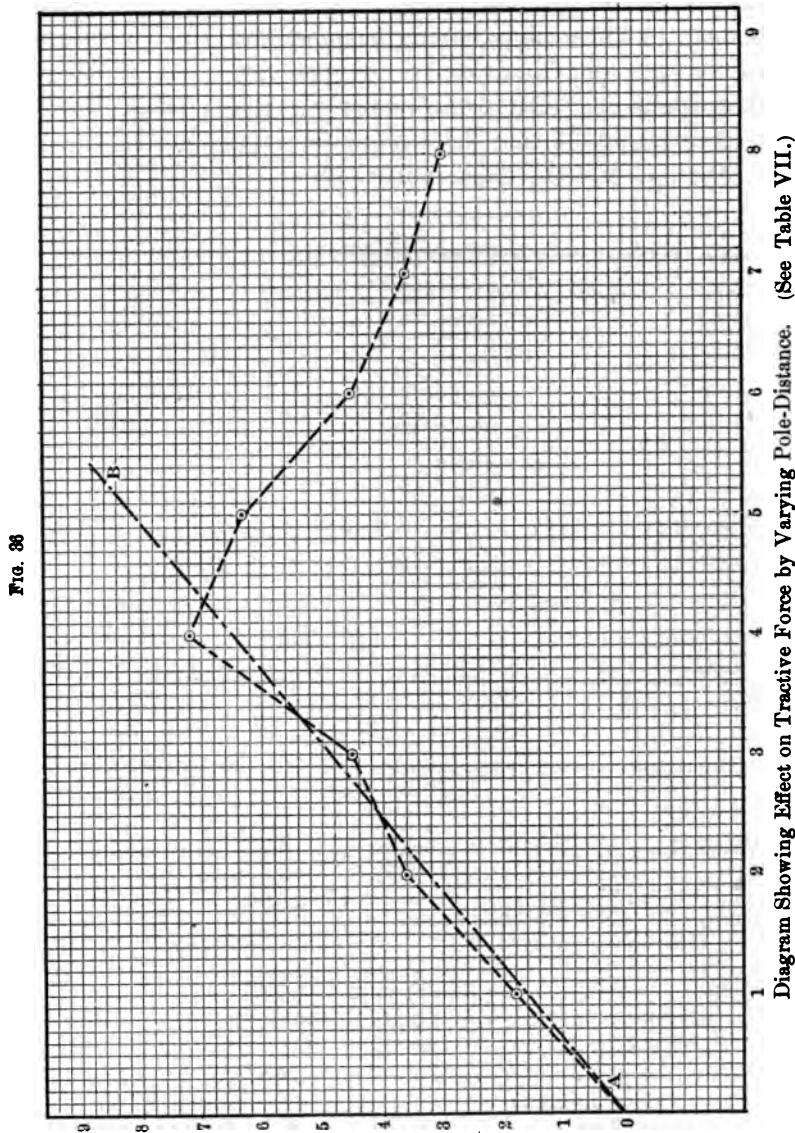
tractive force due to varying flux. The tractive force thus varies directly with the magnetization or flux.

r. Effect of Varying Size, all Other Quantities Being Constant.— Cast-iron was ground and sized between certain limits to deter-

mine the variation of tractive force with reference to size.   The
cast-iron was crushed and reduced to the desired size in a
diamond mortar.   It was found that grains so produced are
rounded, the sharp edges having been broken off.   The crushed
material, as taken from the mortar, was sized between the fol-
lowing limits of meshes to the inch: 190 to 70, 70 to 36, 36 to
12, 12 to 8, and 8 to 5.   Tests were then made on the sizes by
both the rod- and the traction-method.

FIG. 87
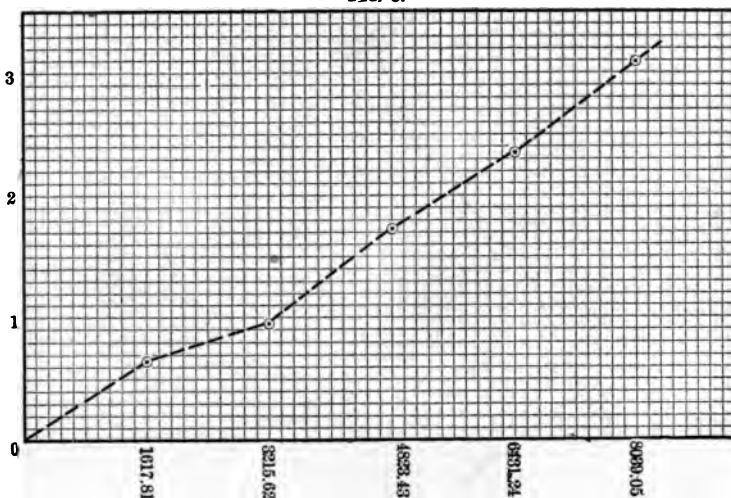
Diagram Showing Variation of Tractive Force With Varying Magnetization.
(See Table VIII.)

With the rod-method the following results were obtained:

TABLE IX.—*Variations in Tractive Force Due to Varying Size of
Material, as Shown by Tests Made on Cast-Iron by the Rod-Method.*

No.	Form.	B (Flux in Iron per 1.2668 Sq. Cms.)	Weight. Grammes.	μ.	H.
1.	Filings, . . .	1452.385	3.84	1.868	777.107
2.	Turnings, . .	1552.138	4.17	1.982	777.107
3.	Unsized, . . .	1617.811	5.20	2.068	777.107
	Below 190-mesh :				
4.	Sized 70-mesh, .	1751.794	5.45	2.241	777.107
5.	Sized 36-mesh, .	1779.592	5.53	2.276	777.107
6.	Sized 12-mesh, .	1806.388	5.72	2.327	777.107
7.	Sized 8-mesh, . .	1885.779	5.74	2.413	777.107
8.	Sized 5-mesh, . .	2019.762	5.78	2.586	777.107
9.	Unsized, . .	2411.716	7.28	3.103	777.107
10.	Rod, . . .	4797.760	12.38	6.120	777.107

From the above table it will be seen that the permeability increases from 2.241 to 2.586 with the sized material. The unsized material passing through a 190-mesh sieve gives a permeability even lower than that of the size of 190 to 70. It is fairly uniform, however, as little fine was made in the mortar. The unsized material, as taken from the mortar, gives a higher permeability than that of the largest size: 3.103 as compared with 2.586.

FIG. 38
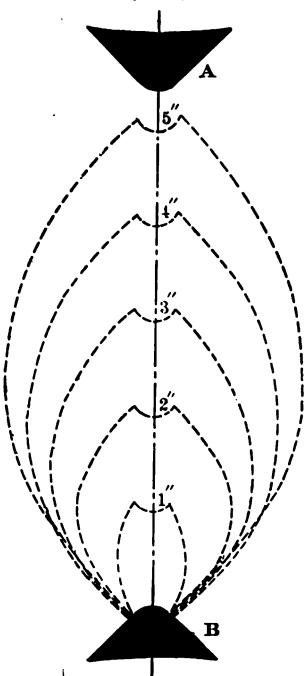
Variations in Divergence of Magnetic Field Due to Varying Pole-Distance.

Filings and turnings were also tested, giving a permeability of 1.868 and 1.982 respectively. The very irregular shape is probably responsible for the drop, which would be produced by a leakage of the flux. The effect of size is also noticeable here—the finer filings having a lower permeability than the larger, tubular-shaped turnings.

Tests were made on a rod turned from the same piece of cast-iron, giving a permeability of 6.120.

The investigation was carried one step further: A wrought-iron rod, a laminated rod and filings from the same rod were

tested by the rod-method, with the following results: For a rod of the best soft Swedish wrought-iron, μ was determined as 7.177; for a laminated rod of Russian sheet-iron, wrapped in spiral form longitudinally, it was 5.887; for filings from the soft iron rod, it was 2.580.

Thus the results obtained from both cast-iron and wrought-iron indicate that both the size and the shape of the material have a marked effect upon its permeability.

The same sizes used in the rod-method were used in the traction-method, and gave similar, although possibly more significant, results.

The tests were made as before, on equal volumes of the sized materials, and the tractive force was measured as already described.

TABLE X.—*Tests, as in Table IX., Made by the Traction-Method Upon Crushed Cast-Iron.*

Mesh.	Weight. Grammes.	Pull. Grammes.
1. Unsized below 5,	2.08	1766.65
2. Sized 8 to 5,	1.85	1561.56
3. Sized 36 to 12,	1.82	1519.57
4. Sized 70 to 36,	1.58	1277.59
5. Sized 190 to 70,	1.52	1124.36
6. Unsized above 190,	1.31	917.94
7. Filings,	0.97	702.36

With this method, as with the rod-method, the unsized material stands highest, then the sized products, next the unsized below 190-mesh to the inch, lastly the filings. This variation is due to the varying percentage of air-space. For sized and unsized materials the per cents. of air-space are 50 and 38 respectively.*

Tests were then made on minerals crushed and sized, as shown in the following table:

TABLE XI.—*Tests, as in Table X., Made Upon Other Minerals.*

No.	Mineral.	Sized Mesh.	Weight. Grammes.	Pull. Grammes.
1.	Pyrite,	100 to 70	1.12	2.250
2.	Pyrite,	70 to 36	1.17	2.150
3.	Pyrite,	70 to 12	1.16	1.925
4.	Pyrite,	12 to 6	1.03	1.650

* P. Rittinger's *Taschenbuch der Aufbereitungskunde,* i., p. 41.

1.	Pyrrhotite,	.	.	.	190 to 60	1.16	5.725
2.	Pyrrhotite,	.	.	.	60 to 30	1.22	4.210
1.	Galena, .	.	.	.	190 to 60	1.82	0.300
2.	Galena, .	.	.	.	60 to 24	1.87	0.260
3.	Galena, .	.	.	.	36 to 12	1.62	0.200
4.	Galena, .	.	.	.	12 to 6	1.55	0.280

The results obtained with the minerals tested are just the reverse of what was obtained for iron, that is, the tractive force increases as the size decreases.

The three minerals chosen, pyrite, pyrrhotite and galena, are fair representatives of the minerals with respect to their magnetic properties. Pyrrhotite is one of the most highly magnetic, and galena one of the least magnetic, of minerals, while pyrite occupies a position midway between them. Pyrrhotite and pyrite give concordant results, while galena follows in a general way, but is more irregular.

The radically different results obtained with cast-iron and with these minerals may be partially accounted for by (a) the higher susceptibility of iron, and (b) the columnar form assumed by it in the magnetic field.

Just how far this columnar form of materials tested would affect the tractive force is difficult to say, but it must have some appreciable effect. The columnar form varies with the size of the material tested.

For minerals, then, it may be said that the tractive force varies with the size—the smaller the size, the higher the tractive force.

s. *Summary.*—From the above determinations the following conclusions may be drawn:

The tractive force varies directly as (1) the mass or weight of mineral (w); (2) the pole-distance (d); (3) the magnetization (F); and (4) the susceptibility (S).*

We may then write

$$T = \frac{S \sigma F w}{K},$$

when σ is the divergence of the field, and K a constant, determined from cast-iron, and used as a basis for comparison.

Solving for S, we have

---

* $S = \dfrac{I}{H}$. I varies directly as d, hence the tractive force (T) varies as S.

$$S = \frac{T K}{\sigma \, F \, w}.$$

But σ varies as the pole-distance d; hence

$$S = \frac{T K}{d \, F \, w}.$$

Substituting this value of S in the formula for permeability, $\mu = 1 + 4\pi S$, we have

$$\mu = \frac{4\pi \, T \, K}{d \, F \, w},$$

and assuming $\mu = 2.068$ (the permeability of cast-iron), and $K = 4.109$, the formula for determining the tractive force will become

$$\mu = 1 + 51.6353 \, \frac{T}{d \, F \, w}.$$

This empirical formula will give the permeability of substances as compared with cast-iron crushed and unsized, below 190 mesh, assuming the permeability of cast-iron to be 2.068 and that of air to be 1.

As previously stated, the results obtained are comparative, being compared with cast-iron. The comparison is based on cast-iron for three reasons: (a) because it was the only available material which could be reduced and sized in a similar manner as the minerals; (b) because it could be tested by both the rod- and the traction-method; (c) because the value of the permeability found could be used to deduce a formula based upon data obtained by the traction-method.

That the value of the permeability thus found is low for highly magnetic substances, has been demonstrated. It may be very near the actual value for the less magnetic substances. For the diamagnetic materials it probably gives a value a little too high.

What the formula is intended to express is the permeability of minerals as compared with a known value of iron reduced to the same physical state as the ores, assuming the permeability of air to be unity.

III. The Relative Magnetic Permeability of Various Minerals.

The table on pages 40 and 41 contains not only the permeability of the minerals and metals tested, but the data from which the calculations were made. In the last column the tractive force expressed in percentage is of the weight of mineral.

The specimens of the minerals named in the above table came from the following localities (where several localities are named, they are placed in the order ·occupied by the corresponding tests in the table):

Magnetite, Port Henry, Essex Co., N. Y.; *Franklinite*, Franklin Furnace, Sussex Co., N. J.; *Ilmenite*, Edge Hill, Montgomery Co., Pa.; *Pyrrhotite*, Stobie mine, Sudbury, Ontario, Canada; and Gap mine, Lancaster Co., Pa.; *Zircon*, Norway; Green River (Hendersonville), Henderson Co., N. C.; and Egansville, Renfrew Co., Ontario, Canada; *Hematite*, Lake Superior; Iron Mt., Minn.; Vermillion Range, St. Louis Co., Minn.; Iron Mt., Mo.; and Cumberland, England; *Corundum*, Jackson Co., N. C.; Gaston Co., N. C.; Lehigh Co., Pa.; East Indies; *Siderite*, Roxbury, Litchfield Co., Conn.; Allevand, France; *Rhodonite*, Franklin Furnace, N. J.; *Limonite*, Nova Scotia; Fleetwood, Berks Co., Pa.; *Pyrolusite*, Thuringia; and Bartow Co., Ga.; *Purite*, French Creek mines, Chester Co., Pa.; Island of Elba; Rio Tinto, Spain; *Manganite*, Bridgeville, Pictou Co., Nova Scotia; *Calamine*, Friedensville, Lehigh Co., Pa.; *Sphalerite*, Freiberg, Saxony; Iowa; Joplin, Jasper Co., Mo.; *Dolomite*, Cumberland, England; Guanajuato, Mexico; Sing Sing, Westchester Co., N. Y.; Berkshire Co., Mass.; *Quartz*, Maine; Kerguelen Island, South Indian Ocean; Chester Co., Pa.; *Rutile*, Magnet Cove, Ark.; Graves Mt., Lincoln Co., Ga.; *Cerussite*, Broken Hills mines, N. S. W.; *Gypsum*, Owasco, Cayuga Co., N. Y.; Derbyshire, Eng.; Grand Rapids, Mich.; *Talc*, Lafayette, Montgomery Co., Pa.; Swain Co., N. C.; Cherokee Co., N. C.; Marietta, Ga; *Molybdenite*, Frankford, Pa.; Tellemarken, Norway; Armidale, N. S. W.; *Cerargyrite*, Silver City, Grant Co., N. M.; *Argentite*, Guanajuato, Mexico; *Magnesite*, Regla, Cuba; Lancaster Co., Tex.; *Orpiment*, Felsobanya, Hungary; *Bornite*, New South Wales; Union Bridge, Carroll Co.,

TABLE X.—*Magnetic Permeability of Various Minerals.*

Mineral.	Weight. (w) Grammes.	Pole-Distance. (d) Centim.	Flux. (F)	Tractive Force. (T) Grammes.	Permeability. (μ)	T in Per Cent. of w.
Iron.............	1.41	4.625	6967.181	1023.741	2.1617	72605.81
Steel.............	1.36	"	"	978.742	2.1532	71966.17
Magnetite......	1.08	"	"	314.712	1.4669	29140.00
Franklinite....	1.16	"	"	297.731	1.4112	23942.15
Ilmenite........	.95	"	"	175.752	1.2871	18500.21
Pyrrhotite......	1.08	"	"	52.742	1.0782	4898.48
"	.99	.828	10924.716	13.454	1.0775	1358.89
Zircon..........	1.04	"	"	5.345	1.0293	513.90
"	.94	.906	10852.724	.758	1.0042	80.21
"	1.08	.828	10924.716	.363	1.0019	33.61
Hematite.......	1.19	"	. "	5.045	1.0242	423.94
"	1.49	"	"	4.365	1.0167	292.95
"	.81	"	"	2.272	1.0160	280.20
"	.81	.921	10835.732	1.876	1.0119	230.86
"	1.61	.828	10924.716	2.302	1.0081	149.90
Corundum......	.82	"	"	3.636	1.0253	443.42
"	.83	"	"	1.218	1.0083	146.74
"	.85	.921	10835.732	.587	1.0035	69.05
"	.80	.828	10924.716	.263	1.0018	32.87
Siderite.........	.87	.921	10835.732	3.941	1.0234	452.98
"	.85	.828	10924.716	3.171	1.0213	373.05
Rhodonite......	.66	.921	10835.732	2.247	1.0176	340.45
Limonite.......	.73	.828	10924.716	1.271	1.0099	174.10
"	.78	"	"	1.345	1.0098	172.43
Pyrolusite......	.73	"	"	1.134	1.0088	155.34
"	.89	"	"	1.218	1.0078	136.85
Pyrite..........	1.03	"	"	1.163	1.0064	112.91
"	1.04	"	"	.400	1.0020	38.46
"	1.02	.906	10852.724	.294	1.0015	28.82
"	1.03	.828	10924.716	.127	1.0007	12.33
Manganite.....	.77	.921	10835.732	.961	1.0061	124.80
Calamine.......	.75	1.000	10450.771	.896	1.0059	119.46
Sphalerite......	.90	.828	10924.716	.909	1.0057	101.00
"	.97	"	"	.309	1.0018	31.84
"	.94	.921	10835.732	.127	1.0007	13.51
Dolomite.......	.64	.828	10924.716	.636	1.0056	99.37
"	.62	"	"	.290	1.0026	46.77
"	.63	.921	10835.732	.225	1.0018	35.71
"	.67	.828	10924.716	.181	1,0015	27.01
"	.67	"	"	.181	1.0015	27.01
Quartz..........	.56	.921	10835.732	.596	1.0055	106.42
"	.64	.828	10924.716	.609	1.0054	95.15
"	.63	"	"	.245	1.0022	38.88
Rutile..........	.90	.921	10835.732	.927	1.0053	103.00
"	.98	.828	10924.716	.836	1.0048	85.30
"	.97	"	"	.518	1.0030	53.40
Cerussite........	1.46	"	"	1.363	1.0053	93.35
"	1.15	.906	10852.724	.410	1.0018	35.65
Gypsum.........	.43	.828	10924.716	.254	1.0033	59.07
"	.51	"	"	.109	1.0012	21.37
"	.49	"	"	.054	1.0006	11.02
"	.48	.921	10835.732	.051	1.0005	10.62
Garnet.........	.76	.828	10924.716	.636	1.0047	83.60
Talc.............	.41	.921	10835.732	.309	1.0039	75.36
"	.44	.828	10924.716	.100	1.0013	22.72
"	.37	"	"	.090	1.0013	24.32